Rheological Parameter Estimation for a Ferrous Nanoparticle-based Magnetorheological Fluid using Genetic Algorithms

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ABSTRACT: This study examines identification of rheological parameters for a constitutive model characterizing the rheological behavior of a ferrous nanoparticle-based magnetorheological (MR) fluid. Particle size is nominally 28 nm and the MR fluid has a weight fraction of 27.5% Fe. A constant shear rate rheometer is used to measure flow curves (shear stress vs. shear rate), as a function of applied magnetic field, of an MR suspension of nanometer-sized iron particles in hydraulic oil. The MR fluid is characterized using both Bingham-plastic (BP) and Herschel–Bulkley (HB) constitutive models. These models have two regimes that can be characterized by a field-dependent yield stress: pre-yield implies that the local shear stress is less than the yield stress, and post-yield implies that the local shear stress is greater than the yield stress. Both models of MR fluid behavior assume that the MR fluid is rigid in the pre-yield regime. However, the post-yield behavior is different. The BP model assumes that the post-yield increase in shear stress is proportional to shear rate. However, the HB model assumes that the post-yield increase in shear stress is proportional to a power law of shear rate. Identification of the model parameters is complicated by model non-linearities, as well as variance in experimental data. The rheological parameters of the BP and HB models are identified using both a gradient-based least mean square minimization procedure, as well as a genetic algorithm (GA). The HB model is shown to represent better, the rheological behavior of the ferrous nanoparticle-based MR fluid. Also, the GA performs better than the gradient-based procedure in minimizing modeling error.

Key Words: magnetorheological fluids, Bingham-plastic, Herschel-Bulkley, genetic algorithms, yield stress, nanoparticles.

INTRODUCTION

MAGNETORHEOLOGICAL (MR) fluids are a class of smart materials whose rheological properties can be varied via application of magnetic field. These fluids are suspensions of soft magnetic particles (such as iron or cobalt) in a carrier fluid (Weiss et al., 1994; Ginder, 1998; Phule and Ginder, 1998). Each ferrous particle has a dipole, the strength of which is proportional to its diameter. Upon application of a magnetic field, these dipoles align parallel to the magnetic field and form chains. A finite stress must develop to yield these chain structures. The field-dependent yield stress of these fluids is continuously and reversibly controllable, which has been the primary impetus driving the implementation of MR fluid-based damping or actuation systems.

The MR fluids have been produced with different types and sizes of magnetic carrier particles. The overwhelming majority of existing MR fluids are composed of micron-scale particles of iron in a non-magnetic carrier liquid (Yang et al., 1986; Bossis and Lemaire, 1991; Lemaire et al., 1995; Bossis et al., 2002; Genc and Phule, 2002). They have yield stress in the range of 20–100 kPa, which is a direct consequence of the solids loading (volume fraction or weight fraction of magnetic particles), and particle size as related to the strength of their magnetic dipoles. However, the density of the iron particles makes them susceptible to settling in the absence of frequent remixing because of the predominance of gravity forces. Once sedimented, residual magnetic attraction between the particles makes re-dispersion difficult (Phule et al., 1999). Larger particles can also lead to abrasion of the components in contact with the fluid. Ferrofluids, which are suspensions of iron particles of size less than 10 nm, have also been reported

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(Butter et al., 2002; Zubarev et al., 2002). However, there is no formation of elongated chain-like microstructures under the application of a magnetic field and they are unable to provide a significant magnetoviscous effect, especially in the context of yield stress. Nanometer-sized iron particles, ranging from 10 to 100 nm, were introduced to reduce settling while maintaining useful yield stress levels (Kormann et al., 1992; Taketomi et al., 1993; Rosenfeld et al., 2002). It was shown that the introduction of nanoparticles reduced settling (John et al., 2002; Trihan et al., 2003), but the yield stresses for MR fluids having constant solids loading was reduced when nanoparticles were substituted for micron scale particles in the MR suspension (Poddar et al., 2004; Chaudhuri et al., 2005). However, the shear stresses in these fluids are comparable with shear stresses achieved in conventional electrorheological (ER) fluids and the phenomenon is also seen to be temperature dependent.

We use the Herschel–Bulkley (HB) and Bingham-plastic (BP) constitutive models to characterize the rheological behavior of the MR fluid. A BP fluid is characterized by two flow regimes, pre-yield and post-yield, and the boundary between these two regimes is characterized by a threshold shear stress or yield stress, τ_y. In the pre-yield regime, where the local shear stress is less than the yield stress, the behavior is assumed to be rigid. Once the shear stress exceeds the yield stress, the MR fluid flows, such that the additional shear stress in excess of the yield stress is proportional to the shear rate via the post-yield viscosity, μ. The HB fluid is also characterized by two flow regimes in the same way as the BP fluid. The pre-yield behavior of the HB fluid is also assumed to be rigid. A HB fluid flows only if the local stress is greater than the yield stress, τ_y. However, in this case, once the shear stress exceeds the yield stress, the MR fluid flows, such that the additional shear stress in excess of the yield stress is proportional via the post-yield consistency, K to a power law of shear rate where the power is denoted by the flow index n. The flow index n can be used to classify the fluid; $n > 1$ indicates a shear-thickening fluid, $n < 1$ indicates a shear-thinning flow, and $n = 1$ indicates a BP fluid. The HB model has been used in several cases to characterize the flow of MR fluids, especially where shear thinning is seen (Lee and Wereley, 1999; Wang and Gordaninejad, 1999).

Genetic algorithms (GA) have been widely used in applications (Wolf and Moros, 1997; Weatherford and Brice, 2003) where a globally optimal solution is required. In conventional estimation methods, a model structure is chosen and the parameters of that model are calculated by minimizing an objective function. The methods typically used for minimization of the objective function are based on gradient descent techniques. These are very susceptible to initial guesses and the obtained parameters may be only locally optimal. On the other hand, GA uses a probabilistically guided search procedure which simulates genetic evolution (Goldberg, 1989; Mitchell, 1998). Populations with stronger fitness are identified and retained, while those with weaker fitness are discarded. The process ensures that successive generations are fitter. The algorithm cannot be trapped in local minima since it employs random mutation procedures. The overall search procedure is stable and robust, and can identify globally optimal parameters of a system.

In this study, we use the rheological data obtained for an MR fluid consisting of nanometer-sized Fe particles of 28 nm mean diameter in a 27.5 wt% suspension of hydraulic oil. We identify the rheological parameters using a simple genetic algorithm (SGA). Constraints were applied within the algorithm to ensure a monotonically increasing trend of the yield stress with an increase in magnetizing current. The obtained parameters provide a good fit to the experimental data. The parameter variations with change in magnetizing current are smooth and physically more meaningful. Comparison of estimation errors with that of a BP model suggests that the HB model is certainly a better choice.

SYNTHESIS OF IRON NANOPARTICLES AND MR FLUIDS

Materials Modification Inc. developed a patented microwave-based process for efficient synthesis of nanopowders. This technique utilizes microwave energy to generate plasma by ionization, disassociation, and recombination of gas molecules. The high temperature vaporizes the precursors, which promotes chemical reactions at the molecular level in the presence of microwaves. These vapors are then rapidly cooled in an inert atmosphere to form powders. Iron powders were synthesized from iron carbonyl precursors according to

$$Fe(CO)_5 \rightarrow Fe(s) + 5CO \uparrow (g)$$

The average particle diameter in these pure iron powders was nominally 28 nm. The magnetic properties of the iron nanoparticles have been reported earlier (Poddar et al., 2004). To prepare stable MR fluids, hydraulic oil was chosen as a carrier fluid. Mobil DTE20 series is used extensively in high-pressure systems including industrial, marine, and mobile services because of their excellent anti-wear properties, multi-metal compatibility, and corrosion resistance. Lecithin was used as a surfactant for producing stable nanofluidic dispersions. Lecithin was mixed in hydraulic oil using a high-speed emulsifier at nearly 11,000 rpm. Iron nanopowders obtained from the microwave plasma synthesis were added to the oil and the mixing continued. Figure 1(a) is a TEM picture of the nano iron particles, from which the nominal particle diameter was found

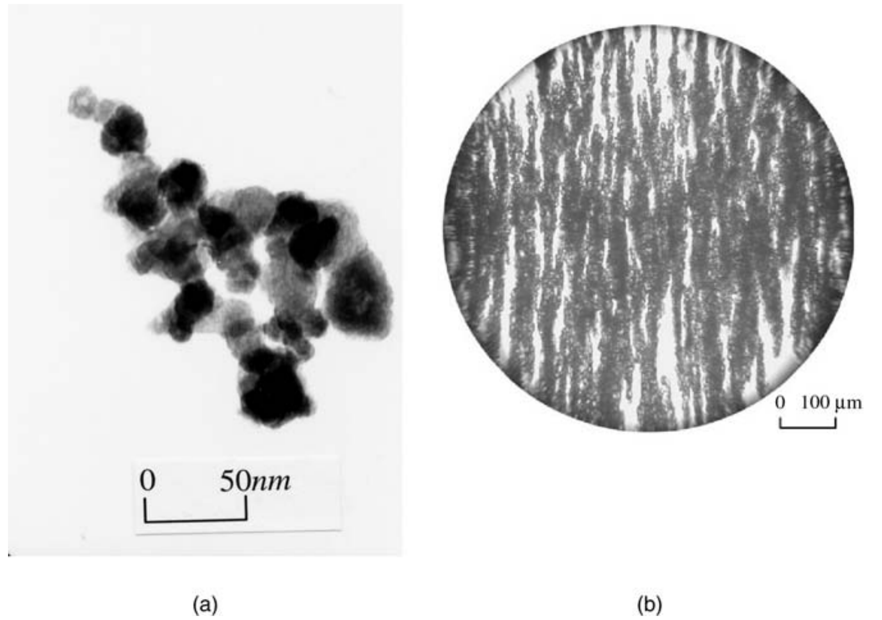

Figure 1. *(a) TEM picture of Fe nanoparticles and (b) optical micrograph of chain formation under the application of a magnetic field.*

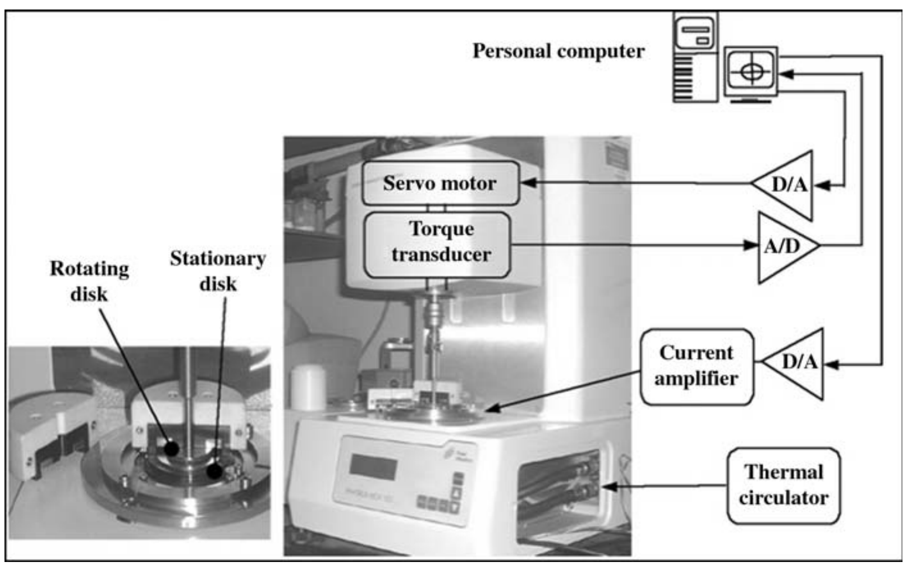

Figure 2. *PaarPhysica MCR300 rheometer with MR cell.*

to be 28 nm. Figure 1(b) shows the formation of chain structures under the application of a magnetic field.

MAGNETORHEOLOGICAL TESTING

The rheological results for this study were obtained using a Paar Physica MCR300 parallel disk rheometer (Figure 2). A standard gap of 1 mm was used to separate the parallel disks. The magnetic circuit is designed so that the magnetic flux lines are normal to the parallel disks. The MR cell is capable of continuously varying the magnetic field applied to the MR fluid sample. The MR cell also included a water-based heating/cooling system to maintain the temperature at 25°C for all the reported data. The top disk rotates while the bottom disk remains stationary. After placing the sample between the plates, the magnetic circuit is closed. As the upper plate rotates, a sensor measures the torque and calculates the corresponding force exerted on the moving plate. The shear stress at a designated point on the plate is then evaluated. A shaft encoder measures the angular rate and the corresponding shear rate.

In these tests, a 0.3 mL sample of MR fluid was placed between the two parallel disks and rotation of the upper disk was accomplished via shear rate control. The range of shear rates tested was from 0.1 to $1500\,s^{-1}$. The maximum shear rate was limited to $1500\,s^{-1}$ because above this speed the fluid was expelled from between the disks. For each fluid sample, 20 measurements

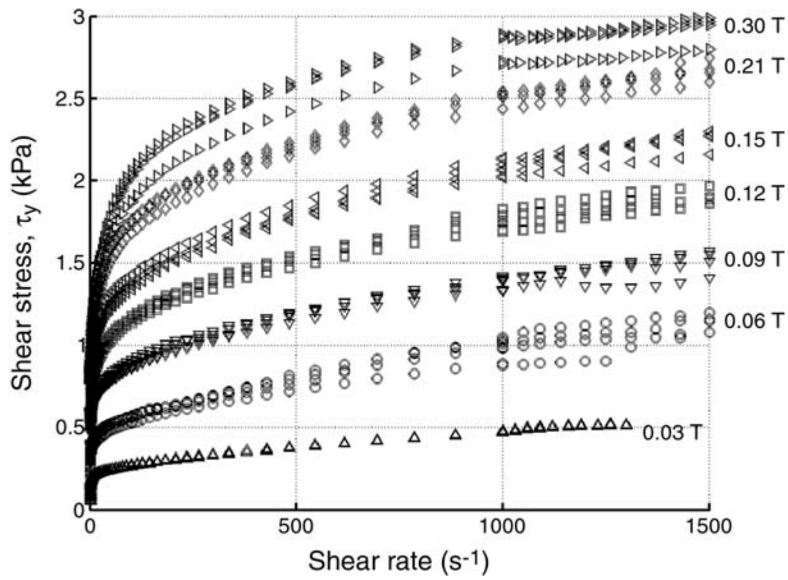

Figure 3. *Rheological curves.*

were taken from 0.1 to $10\,s^{-1}$, 20 points between 10 and $100\,s^{-1}$, 20 points between 100 and $1000\,s^{-1}$, and 10 points from 1000 to $1500\,s^{-1}$. The only exception was for a relatively low current of 0.2 A, when the fluid was expelled from between the disks for shear rates above $1300\,s^{-1}$. The measurements were taken over a range of currents from 0.2 to 2 A, in steps of 0.2 A. For each test, the shear rate was held constant for 5 s until the measured shear stress reached a nominally steady-state value, in order to ensure consistency in the measurements. From these constant rpm tests, a steady-state flow curve for shear stress, τ, versus shear rate, $\dot{\gamma}$, was produced by the rheometer. Figure 3 is a plot of the flow curve data obtained from these tests at different magnetization levels.

RHEOLOGICAL MODELS

The MR fluids demonstrate nonlinear behavior when subjected to external magnetic fields. The rheological behavior of these materials can be separated into distinct pre-yield and post-yield regimes. MR fluids are observed to exhibit a strong field-dependent shear modulus and a yield stress that resists flow until the shear rate reaches a critical value. The BP model has been used as a constitutive model for these fluids (Genc and Phule, 2002). The simplicity of this two-parameter model, with yield stress, τ_y, and post-yield viscosity, μ, has led to its wide use for representation of field-controllable fluids. This model assumes that the fluid exhibits shear stress proportional to shear rate in the post-yield region and is described by the following equation:

$$\tau = \tau_y + \mu\dot{\gamma} \tag{1}$$

However, in cases where the fluid experiences post-yield shear thickening or shear thinning, the post-yield viscosity is no longer constant, but varies with shear rate. The HB model is more suitable as a constitutive model for MR fluids (Lee and Wereley, 1999) and has been applied to analysis of MR-based devices (Wang and Gordaninejad, 1999, 2001; Lee et al., 2002). The HB model is a modification of the equation for a power law fluid and the equation is given below:

$$\tau = \tau_y + K(\dot{\gamma})^n \tag{2}$$

where yield stress, τ_y, consistency, K, and flow index, n, are the model parameters. The model assumes that below the critical value of stress (τ_y), the suspension behaves as a rigid solid. The flow index number, n, characterizes the post-yield behavior; $n > 1$ indicates shear thickening and $n < 1$ indicates shear thinning. The BP model is a special case ($n = 1$) of the HB model.

GENETIC ALGORITHM FOR PARAMETER ESTIMATION

The problem of nonlinear parameter estimation has attracted considerable interest. The use of sum of squared error as the objective function leads to a quadratic minimization problem. Most algorithms perform a minimization of the cost function starting at a user-provided initial guess. They make use of additional information, usually function gradients, to approach the minima. However, such techniques only yield a local minimum in the proximity of the initial case. In case of nonlinear estimation, there may exist multiple optimal solutions that are physically equally significant and identification of all solution sets is not possible unless the optimizer is run with different initial guesses. The availability of function gradients at all points is also not always possible or is computationally costly.

The GAs are a potential tool for finding global solutions in large parameter spaces since many different solution sets are investigated and refined simultaneously to identify near-optimal solutions rather than a single solution. GAs are different from usual optimization and search procedures (Goldberg, 1989; Mitchell, 1998) as they search from a population of points and use probabilistic transition rules instead of deterministic ones. An important consideration is that GA uses only objective/fitness function information, not derivative or any other auxiliary knowledge. The evolution of a generation in order to obtain a new generation consists in selecting best individuals in order to be a member of the new generation, and in adding to this generation the individuals which are the result of crossing over and mutation of the selected members. Convergence is guaranteed by the selection that makes the best solution of the new generation better or equal to the best solution of the previous generation. The overall search procedure is stable and robust and can identify globally optimal parameters of a system. GAs have the ability to solve highly nonlinear functions where other techniques fail.

We have used a SGA (Srinivas and Patnaik, 1994; Man et al., 1996; Renders and Flasse, 1996) for estimating the parameters of the rheological model. A population of P individuals is generated, each composed of multiple binary strings of length N. The length of each individual depends on the number of parameters to be estimated; $2N$ for the BP model and $3N$ for the HB model. Each individual string is then mapped onto a rheological parameter using maximum and minimum bounds as follows:

$$\text{Parameter value} = \frac{(\text{upperbound} - \text{lowerbound})/(2^N - 1)}{(2^N - 1)} \times (\text{decoded value of binary string}) \tag{3}$$

The initial population is generated randomly and spread over the entire space of possible parameter values. The model Equation (2) is then used to determine the error, and corresponding fitness value, of each population member. For every generation, we apply three genetic operators (reproduction, crossover, and mutation) to create a new generation. Reproduction is simulated by a simple roulette wheel-based selection scheme. Crossover is carried out at a single point for each parameter of the model. In order to prevent the algorithm from getting stuck in local minima, new genetic information is periodically injected by mutation. The objective function used is the sum of squared errors and the corresponding fitness is evaluated as the reciprocal of the objective function. For each individual,

$$\begin{aligned} &\text{Squared error } E_j = \sum_{i=1}^{N_e} w_i \times (\tau_i - \hat{\tau}_i)^2 \\ &\text{Fitness value } F_j = \frac{1}{E_j} \end{aligned} \tag{4}$$

where N_e is the number of experimentally obtained points for each magnetizing current, $\hat{\tau}_i$ is the measured shear stress for a particular strain rate, and τ_i is the shear stress at the same strain rate for a particular set of rheological parameters. w_i is used as a weighing factor. The search procedure is carried out for all values of magnetizing current. The flow index number (n) is allowed to vary from 0 to 1. In case of the yield stress (τ_y), we select the highest measured shear stress as the upper bound and the estimated yield stress at the previous magnetization as the lower bound, resulting in a model with monotonically increasing yield stress. This constraint was included because the theory of increasing yield stress with higher magnetization is physically more feasible.

Several termination criteria for a genetic search have been proposed. One simple criterion is to stop the search when almost all individuals in the population are identical (or nearly so); another criterion is to test the improvement in the best fitness score over successive generations. However, the first criterion can lead to extensive search times while the second one is not good for functions characterized by 'plateau-type' regions (Salomon, 1998). In our work, we stopped the GA after a fixed number of generations, which was chosen as a compromise between objectives for convergence, computing time, and accuracy.

RESULTS

The parameters of the constitutive models are obtained after carrying out global optimization using a SGA. The algorithm was run with a population size (P) of 2000 and using 30 bits (N) for encoding each rheological parameter. The probability of crossover (P_c) is 0.85 and the probability of mutation (P_m) is chosen as 0.025. Constraints have been applied to the search scheme to ensure that a monotonically increasing characteristic was obtained for yield stress (τ_y) with increasing magnetic flux density. The cost function is evaluated using Equation (4). The weighing factor, w_i, was adjusted so that the algorithm emphasized a better fit to the data at higher shear rates rather than at low shear rates.

The parameters identified for the BP model are shown in Figure 4. The yield stress is seen to vary from 280 Pa

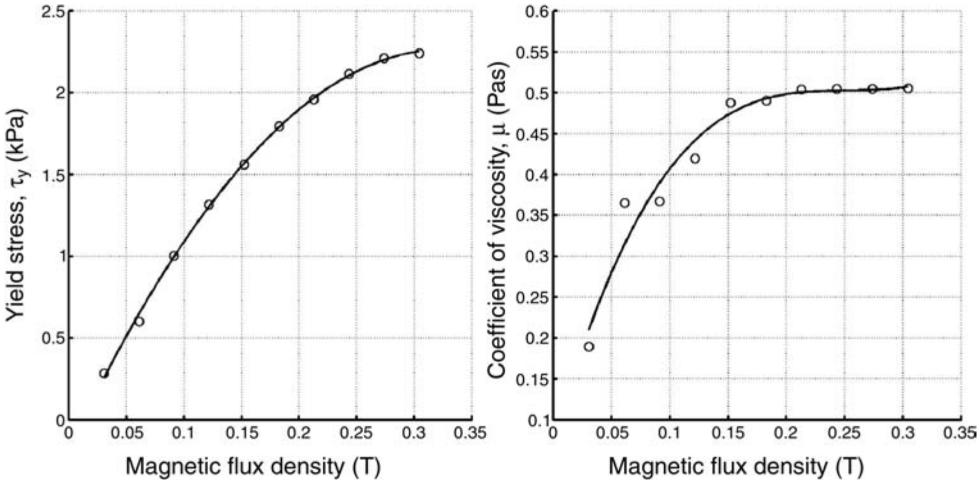

Figure 4. *Parameters of BP model.*

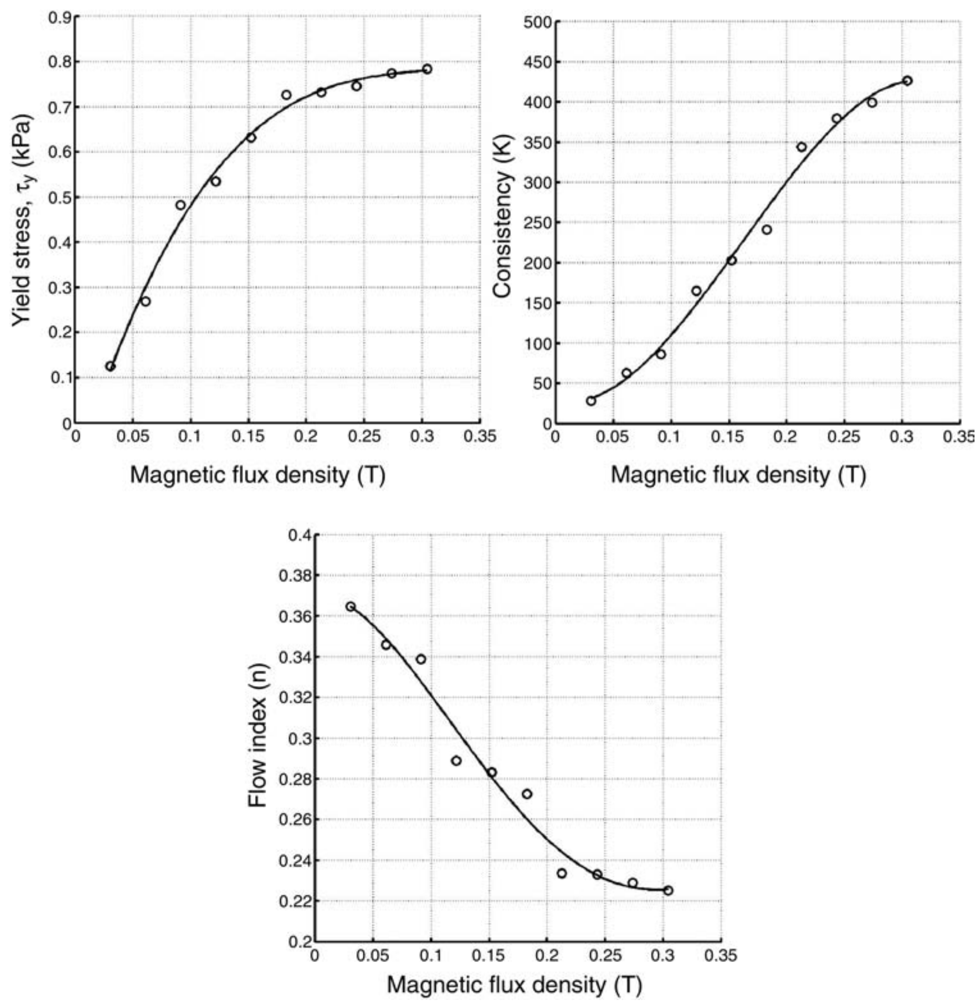

Figure 5. *Parameters of HB model.*

at 0.03 T to 2250 Pa at 0.30 T. The variation in yield stress is smooth and monotonically increasing. The viscosity of the MR fluid also increases as magnetic field increases, and reaches a plateau-value of 0.5 Pa s at a magnetic flux density of 0.2 T.

The parameters identified for the HB model are shown in Figure 5. The parameters change smoothly over the range of magnetic flux densities tested. Yield stress (τ_y) and consistency (K) show an increasing trend and saturate at higher flux densities. The values

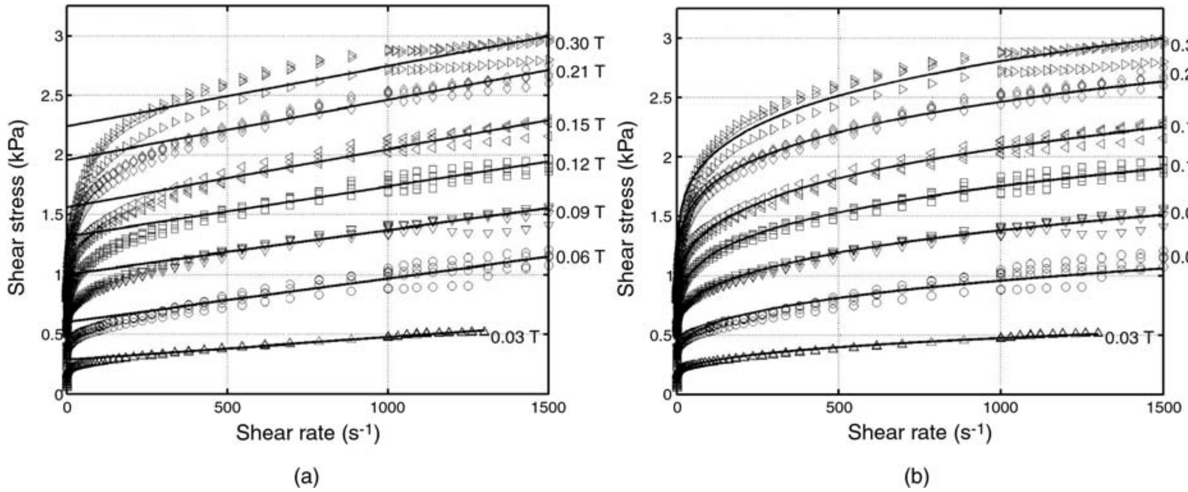

Figure 6. *Rheological flow curves with fitted models: (a) BP model and (b) HB model.*

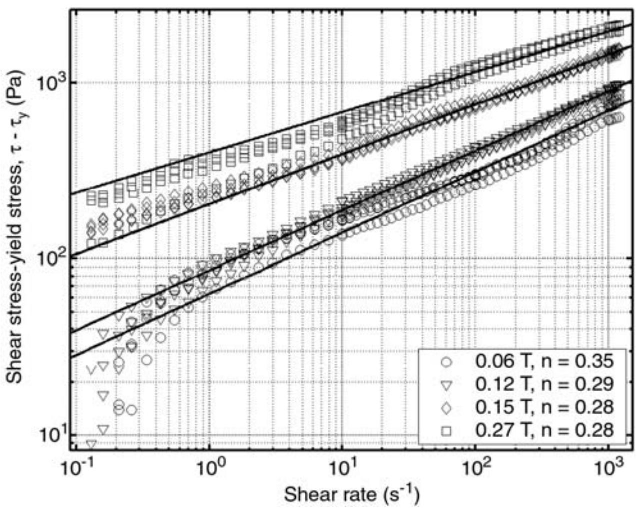

Figure 7. *HB model fitting in logarithmic scale.*

of field-dependent yield stress vary from 120 to 790 Pa and are lower than those obtained from the BP model. The flow index (n) constantly maintains a value less than unity, which indicates that the post-yield flow is non-linear and shear thinning. It is notable that unlike the constant flow index characteristic of nanocobalt-based MR fluids (Chaudhuri et al., 2005), the flow index for nanoiron-based MR fluids decreases steadily with increasing magnetization.

Figure 6 shows plots of the experimental data with the superimposed BP and HB models. The identified parameters are used in Equations (1) and (2) to compute the fitted curves. The BP model fits the data well at high shear rates, but is unable to capture very low shear rate behaviors. The HB model provides a much better fit over the entire range of measured data; a log–log plot of the model fit is shown in Figure 7 (only four cases are shown for clarity). The robustness of the search scheme is evident since it is able to fit the data even in the presence of a high amount of scatter.

In order to compare parameter identification results, we also carried out parameter identification using a conjugate gradient-based scheme. The same cost function was used and the minimization carried out using the *fmincon* function in Matlab, with similar constraints. The identified parameters exhibited similar trends and values as a function of magnetic flux density. To check the efficacy of the HB model, we compare the modeling errors with those obtained by using BP model (Figure 8). The percentage error is evaluated as the absolute error compared to the peak value of shear stress measured at a particular level of magnetization. The BP model presents an average error of 7.5%, while the error was less than 2% when a HB model is used. The use of GA is also seen to produce more accurate results than the gradient-based (LMS) method.

CONCLUSIONS

In this study, we have identified the parameters of two constitutive models used to characterize the rheological properties of a ferrous nanoparticle-based magneto-rheological (MR) fluid. Tests were conducted on a suspension of iron particles, approximately 28 nm in diameter, in silicon oil at different levels of magnetizing current. The applied magnetic flux density is varied from 0.03 to 0.30 T and the rheological measurements are obtained. A simple genetic algorithm (SGA) is used to estimate the parameters of the constitutive model. Constraints are imposed on the minimization process to ensure monotonic increase of the yield stress with increasing magnetization, while keeping the other parameters unconstrained. The following points are noted:

(i) The maximum shear stress in the fluid is seen to vary from about 0.5 to 3 kPa, which is far

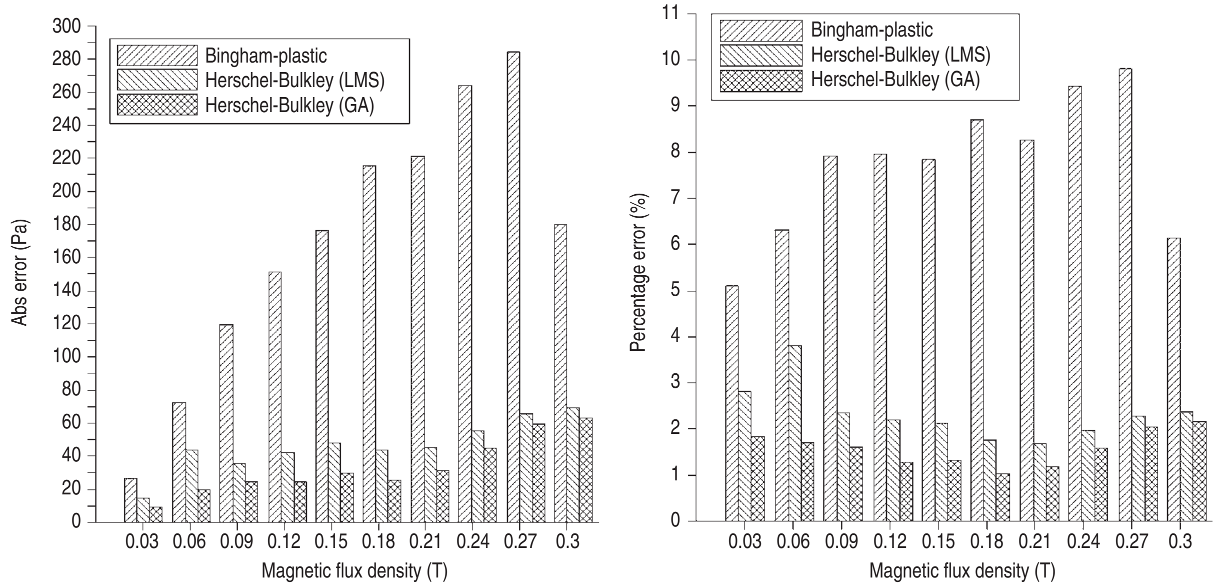

Figure 8. *Errors in model fitting.*

lower than the stresses obtainable in ferrous MR fluids with micron-sized particles. The rheological curves also show considerable shear-thinning effects.

(ii) The dynamic yield stress is seen to vary from about 0.1 kPa at 0.03 T to almost 0.8 kPa at 0.30 T for the HB model, while the same parameter takes values ranging from 0.25 to 2.25 kPa for the BP model.

(iii) The corresponding flow index decreases from 0.36 to 0.23, indicating shear-thinning effects in the fluid.

(iv) A notable feature is the manifestation of saturation at higher values of magnetic flux densities, which implies that higher yield stresses cannot be obtained simply by raising the magnetic flux density throughout the sample.

(v) The percentage absolute errors in model fitting are limited to 2% of the peak shear stress observed for a particular level of magnetization when GA is used for the HB model, and are lower than the corresponding errors when an LMS technique is applied.

(vi) It was shown that the HB model well represents the observed data over a much larger shear rate range compared to the BP model. Quantitatively, the BP model has errors from 5% to 9.5%, which is more than three times the error observed for the HB model.

Thus, the Herschel–Bulkley model is a better choice as a constitutive equation for describing the behavior of ferrous nanoparticle-based MR fluids over a specified range of shear rate. Genetic algorithms can also be efficiently applied to such rheological parameter estimation problems and provide results that are comparable to those obtained from conventional gradient-based schemes.

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